1. 200 crosses and 180 noughts are written on a circle; the number of pairs of neighboring crosses is equal to 50. What is the number of pairs of neighboring noughts?
Answer: 30.
2. Three bells begin to strike at the same time. Intervals between the strokes of bells are $4/3$ seconds, $5/3$ seconds and 2 seconds, respectively. The strokes happening at the same time are perceived as one. How many strokes will be heard during 1 minute? (including the first stroke and the last one).
Answer: 85.
3. Two players are playing the game, consisting of the following. Written the numbers: 0, 1, 2, ..., 1024. The first player crosses out 512 numbers of his choice, the second crosses out 256 from the remaining numbers; then again the first one crosses out another 128 numbers, and then the second crosses out 64 numbers, etc. On his last fifth move, the second player crosses out one number. There remain two numbers, and the second player pays the difference between these numbers to the first one. How much will the second player pay the first one, if both play their best?
Answer: 32.
4. The sum of the natural numbers a_1, a_2, ..., a_{10} is equal to 1001. Find the largest possible value of the greatest common divisor of these numbers.
Answer: 91.
5. Find the smallest natural number $n > 1$, for which the sum of squares of consecutive natural numbers from 1 to n should be the square of a natural number.
Answer: 24.
6. How many rational terms are there in the expansion $\left(\sqrt{2}+\sqrt[3]{3}\right)^{100}$?
Answer: 17.
7. How many different pairs of integers x and y from 1 to 1000 are there, for which $\frac{x^2+y^2}{49}$ is an integer? (Pairs (x,y) and (y,x) are considered to be the same).
Answer: 10153.
8. Find the sum of the roots of the equation $\left[\frac{x}{2}\right]^2+\frac{2}{[x]}=5x$. Here $[x]$ is the largest integer not greater than x.
Answer: $19,82$.
9. It is known that among the roots of the equation $x^3+ax^2+2017x+2018=0$ there are two numbers the sum of which is equal to zero. Find the coefficient a.
Answer: $\frac{2018}{2017}$.
10. Find the least value of the parameter p, for which the equation
$\lg\left(x^2+2px\right)-\lg\left(8x-6p-3\right)=0$ has the unique solution.
Answer: $-\frac{1}{2}$.
11. A car starts from point O and rides with a constant velocity of 70 km/h along a rectilinear highway. A cyclist who is at a distance of 16 km from point O, and at a distance of 3 km from the highway, wants to deliver a letter to the driver of the car. What is the minimum speed (km/h) with which the cyclist should ride to fulfill his intention?
Answer: $13\frac{1}{8}$.
12. 19 points are given on a circle and all possible chords connecting these points are drawn. It is known that no three of the drawn chords intersect at one point. How many parts is the disk divided into?
Answer: 4048.

13. The diagonals of two identical cubes of edge length equal to 8 lie on the same straight line. The vertex of the second cube coincides with the center of the first one, and the second cube is rotated by $60°$ about the diagonal in relation to the first one. Find the volume of the common part of these cubes.
Answer: 72.
14. On the sphere with radius of 2 there are three pairwise tangent circles with radius of 1. Find the radius of the smallest circle located on this sphere and tangent to these three given circles.

Answer: $1-\sqrt{\dfrac{2}{3}}$.
15. A plane is drawn through every three vertexes of a cube, which lie at the ends of every three edges converging at one vertex. Find the volume of the solid bounded by these planes if the edge of the cube is equal to 3.

Answer: $\dfrac{9}{2}$.
16. What is the largest area of the projection of a rectangular cuboid with dimensions 1, 2, 3 onto a plane?
Answer: 7.
17. One of the sides of a square lies on the line $y=2x-17$, and other two vertices are on the parabola $y=x^2$. Find the smallest area of the square.
Answer: 80.

18. Find $\lim\limits_{x\to 1}\left(\dfrac{2018}{1-x^{2018}}-\dfrac{1514}{1-x^{1514}}\right)$.
Answer: 252.

19. Find $\lim\limits_{x\to+\infty}\ln\left(1+2^x\right)\ln\left(1+\dfrac{3}{x}\right)$.
Answer: $3\ln 2$.
20. Given a differentiable function $f:[0,1]\to\mathbb{R}$. It is known that
$x\sin f(x)+x^2=f(x)\sin x+2f^2(x)$ for all $x\in[0,1]$. Find $\left|f'_+(0)\right|$ (here $f'_+(0)$ is the right-hand derivative at the point 0).

Answer: $\dfrac{1}{\sqrt{2}}$.
21. The hour hand has a length of 2 centimeters, and the minute hand is $2{,}2$ centimeters long. The angle between the hands varies at a constant speed. Find the distance between the ends of the hands at the moment when it changes fastest.

Answer: $\dfrac{\sqrt{21}}{5}$.

22. Evaluate the integral $\displaystyle\int_0^{2\pi} e^{\cos x}\cos(\sin x)\cos 5x\,dx$.

Answer: $\dfrac{\pi}{120}$.

23. Evaluate the sum of the integrals $\displaystyle\int_{\sqrt{\pi/6}}^{\sqrt{\pi/3}}\sin\left(x^2\right)dx+\int_{1/2}^{\sqrt{3}/2}\sqrt{\arcsin x}\,dx$.

Answer: $\dfrac{\sqrt{\pi}\left(\sqrt{6}-1\right)}{2\sqrt{6}}$.

24. A tank contains 100 liters of solution with 10 kg of dissolved salt. Water enters the tank at a rate of 5 liters per minute, and the mixture drains at the same rate into another 100 -liter tank, originally filled with clean water. Surplus liquid pours out from it. What is the greatest amount of salt (in kilograms) in the second tank?

Answer: $\frac{10}{e}$.

25. Let $x^* = x^*(t)$ be the solution to the differential equation $t^2\ddot{x} - 2t\dot{x} + 2x = 0$, $0 \le t \le 1$, that satisfies the boundary conditions $x(t) = o(t)$ when $t \to 0$, $x(1) = 3$. Find $x^*(0{,}2)$.

Answer: $\frac{3}{25}$.

26. The arc of the curve $\rho = \frac{\sin\varphi - \cos\varphi}{1 + \sin 2\varphi}$, $\frac{\pi}{4} \le \varphi \le \frac{\pi}{2}$, rotates around the ray $\varphi = \frac{\pi}{4}$. Find the volume of the solid of revolution.

Answer: $\frac{\pi\sqrt{2}}{20}$.

27. Find $\lim\limits_{n\to\infty} \iint\limits_{E_n} \sin\left(x^2 + y^2\right) dxdy$, where $E_n = \left\{(x,y) \in \mathbb{R}^2 \,\middle|\, |x| < n,\ |y| < n\right\}$.

Answer: π.

28. Find the area of the region, bounded by the curve given by the equation $(2018x + 2y - 1)^2 + (2018x - 3y + 2)^2 = 1$.

Answer: $\frac{\pi}{10090}$.

29. Find the sum of the series $\sum\limits_{n=1}^{\infty} \frac{4^n\left((n-1)!\right)^2}{(2n)!}$.

Answer: $\frac{\pi^2}{2}$.

30. Evaluate the integral $\int\limits_{-\infty}^{+\infty} \frac{dx}{\left(x^2 + 1\right)^3}$.

Answer: $\frac{3\pi}{8}$.