1. Square cells of 7×8 rectangle are painted in white, blue and red colors so that it is possible to find at least one cell of each color in any 2×2 square. Find the greatest possible number of red cells.
Answer: 32.
2. 45 students from different cities participated in the Mathematical Olympiad. At the end of the Olympiad n pairs of students exchanged email addresses. After some time, Misha from Moscow wanted to get the email address of Masha from Mogilev. At which smallest value of n we can assuredly state, that Misha will be able to find out Masha's email (through other students)?
Answer: 947.
3. 30 teams participated in the football tournament. At the end of the competition it turned out, that in any group of three teams it is possible to single out two teams which scored equal points in three games within this group (3 points are given for the victory, 1 point — for the draw, 0 — for the defeat). What is the least possible number of draws that can occur in such a tournament?
Answer: 135.
4. Natural number n is such that number $2n$ has 28 natural divisors, and number $3n$ has 30 natural divisors. How many natural divisors does number $6n$ have?
Answer: 35.
5. How many solutions does the equation $\left[\frac{x}{2}\right]+\left[\frac{x}{3}\right]+\left[\frac{x}{6}\right]=x$ have in the segment $\left[1,2015\right]$, $\left[x\right]$ being the integer part of x?
Answer: 335.
6. Find the greatest solution of the equation $x^{x}=10^{10^{1003}}$.
Answer: 10^{1000}.
7. It is known that none of two diagonals of the convex 17-gon are parallel and none of three diagonals intersect in one point. Find the number of those points of intersection that lie outside the given 17-gon.
Answer: 3094.
8. How many ways can be used to select from numbers $1,2,\ldots,12$ the group of three or more numbers in which none of two numbers differ by 6? (The order of numbers in the group is not important).
Answer: 656.
9. Four balls of radius $\sqrt{3}$ lie on the horizontal plane. Their centers are in the vertices of the square with the side length of $2\sqrt{3}$. The fifth ball of the same radius is placed from above in the hole formed by these balls. Find the distance from the highest point of the fifth ball to the plane.
Answer: $2\sqrt{3}+\sqrt{6}$.
10. A glass of cylindrical shape, filled to the brim with water, stands on a horizontal plane. The height of the glass is twice the diameter of its base. At what angle φ $(0\le\varphi\le\frac{\pi}{2})$ from the vertical line should the glass be tilted, so that exactly one third of the water content would pour out?
Answer: $\operatorname{arctg}\frac{4}{3}$.
11. Two diagonal lines were drawn in the trapezium and their middles were connected. Together with the lower base the obtained segments form a trapezium again. This operation was repeated 2015 times. The upper base of the final trapezium turned out to be equal to the top base of the initial trapezium. The height of the initial trapezium is equal to $\sqrt{2}$. Find the area of the initial trapezium if the length of its top base is equal to 4.
Answer: $8\sqrt{2}$.
12. The 10×10 table is filled with integer numbers from 1 to 100. Number 1 is located in any cell of the table, number 2 belongs to the row, the ordinal number of which is equal to the number of the column, to which 1 belongs, number 3 is in the row, the number of which coincides with the number of the column containing number 2, etc. Finally, number 100 is in the row, the ordinal number of which coincides with

the number of the column, to which number 99 belongs. How much does the sum of numbers of the row containing number 1 differ from the sum of numbers of the column containing number 100?
Answer: 90.
13. Calculate the determinant

$$\begin{vmatrix} 1 & 2 & 3 & \dots & 9 & 10 \\ 10 & 1 & 2 & \dots & 8 & 9 \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 2 & 3 & 4 & \dots & 10 & 1 \end{vmatrix}.$$

Answer: $-5,5 \cdot 10^9$.

14. The matrix $A = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$ is given. Find the sum of all elements of the matrix $\cos A$.

Answer: $2,5$.

15. Find the smallest value of the expression $\sqrt{(x-9)^2+4}+\sqrt{x^2+y^2}+\sqrt{(y-3)^2+9}$.
Answer: 13.

16. Find the largest value of the expression $\left|\sum_{k=1}^{2015} \sin 2x_k\right|$ given that $\sum_{k=1}^{2015} \sin^2 x_k = 4$.

Answer: $4\sqrt{2011}$.

17. The inclined line, passing through the point $(2015;0)$, intersects the hyperbola $y = \dfrac{10}{10+x}$ in points A and B. Find the abscissa of the middle of the segment AB.
Answer: $1002,5$.

18. Let A_0, A_1, A_2, A_3, A_4 be consecutive vertices of a regular pentagon inscribed in the circle of radius 1. Find $A_0A_1 \cdot A_0A_2$.

Answer: $\sqrt{5}$.

19. Find $\sum_{n=1}^{\infty} \dfrac{\sin 3n}{n}$.

Answer: $\dfrac{\pi-3}{2}$.

20. Find $\sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \dfrac{m^2 n}{3^m (n \cdot 3^m + m \cdot 3^n)}$.

Answer: $\dfrac{9}{32}$.

21. Find $\lim_{n\to\infty}\left(\dfrac{1}{n+1}+\dfrac{1}{n+2}+\dots+\dfrac{1}{2n}\right)$.

Answer: $\ln 2$.

22. Find $\lim_{n\to\infty}\sqrt[n]{a_n}$, if $a_1 = 19$, $a_2 = 361$, $a_n = \sum_{i=1}^{n-1} a_i a_{n-i}$ with $n \ge 3$.

Answer: 76.

23. Find $\lim_{x\to 0}\left(\frac{\sqrt{1-x^2}}{\cos x}\right)^{\frac{1}{x^4}}$.

Answer: $\frac{1}{\sqrt[6]{e}}$.

24. It is known that the limit $\lim_{x\to 0}\frac{1}{4bx-\sin x}\int_0^x \frac{t^2}{\sqrt{t-a}}dt$ is finite and is not equal to zero. Find the value of b.

Answer: $\frac{1}{4}$.

25. Calculate $\int_0^{\pi/2}\frac{\sin^{2015}x}{\sin^{2015}x+\cos^{2015}x}dx$.

Answer: $\frac{\pi}{4}$.

26. Calculate $\int_0^{+\infty}(x-\frac{x^3}{2}+\frac{x^5}{2\cdot 4}-\frac{x^7}{2\cdot 4\cdot 6}+\ldots)(1+\frac{x^2}{2^2}+\frac{x^4}{2^2\cdot 4^2}+\frac{x^6}{2^2\cdot 4^2\cdot 6^2}+\ldots)dx$.

Answer: $\sqrt{e}$.

27. Function $y=y(x)$ satisfies the differential equation $y''-\frac{y'}{x}+4x^2y=0$ and the conditions $y(0)=1$, $y(\sqrt{\frac{\pi}{2}})=2$. Find the value of $y(\sqrt{\frac{\pi}{4}})$.

Answer: $\frac{3\sqrt{2}}{2}$.

28. The reservoir is filled with 75 liters of water containing 3 kg of dissolved salt. The inflow of fresh water (containing no salt) is 4 liters per minute, and the flow of the mixture from the reservoir is 2 liters per minute. The concentration is kept uniform by mixing. Find the mass of salt (in kg), which will be contained in the reservoir in 25 minutes

Answer: $1,8$.

29. Polynomial $P(x)$ satisfies the identity

$$x^{2015}+x^{2014}+x^{2013}+1\equiv(x^3+x^2+x+1)\cdot P(x).$$

Find $P(-1)$.

Answer: 1007.

30. It is known that infinitely differentiable function $f:\mathbb{R}\to\mathbb{R}$ satisfies the identity $f(x+y)\equiv f(x)+f(y)+2xy$. Find $f(2015)$, if $f(1)=-2015$.

Answer: -2015.