1. How many times during 24 hours the angle between the hour and the minute hand is exactly $38°$?
Answer: 44.
2. A tamer of wild animals wants to bring 5 lions and 4 tigers to the circus arena so that there won't be two tigers following each other. How many ways can he arrange the animals?
Answer: 43200.
3. The regular 17-sided convex polygon $A_1A_2...A_{17}$ is given in a plane. How many obtuse triangles $A_iA_jA_k$ are there?
Answer: 476.
4. For every pair of integers (x, y), satisfying the equation

$$(x^2+y^2)(x-2y+15)=2xy,$$

calculate the sum $x+y$. Write the greatest of these sums in the answer.
Answer: 28.
5. Find the sum of the roots of the equation $1-|x+1|=\frac{[x]-x}{|x-1|}$, where $[x]$ means the largest integer not greater than x.
Answer: $-2-\sqrt{5}$.
6. The first 2014 natural numbers are placed in order along the circle. Then every second number is sequentially crossed out. This process lasts till only one number is left. What is this number?
Answer: 1981.
7. It is known that for the function f, defined on the set of natural numbers, $f(m)\neq f(n)$, if $|m-n|$ is a prime number. What is the least number of values of this function?
Answer: 4.
8. Red and green points are marked on a straight line, there being at least one point of each color. The points, between which there are exactly 10 or exactly 15 other points, have the same color. What the greatest number of points can be marked?
Answer: 26.
9. In a a trapezium $ABCD$ the bases $BC=20$, $AD=30$ and the lateral sides $AB=6$, $CD=8$ are given. Find the radius of the circle, crossing the points A and B and touching the side CD.
Answer: 15.
10. The cube is inscribed in the right circular cone with the radius of base 1 and height 3 so that the base of the cone contains one of the faces of the cube, and the vertices of the opposite face are touching the surface of the cone. Find the edge of the cube.
Answer: $\frac{9\sqrt{2}-6}{7}$.
11. The bee hive is located in the center of an equilateral triangle with the side length of 1. One side of the triangle is covered with honey, the other — with jam, and the third one is sprinkled with sugar. A bee should fly out of the hive, eat honey, jam and sugar and return home. Find the length of its shortest path.
Answer: $\frac{\sqrt{21}}{3}$.
12. From the three points, located at the distance of 36 m, 72 m и 108 m from the base of the water tower, the tower is seen at the angles, the sum of which is equal to $90°$. Find the height of the tower (in metres).
Answer: 36.
13. Two trains started at the same time from point A to point B. The motion of each train was uniformly accelerated first (accelerations of the trains are different, initial speeds are equal to zero), and then, after the train has reached certain speed, the motion was uniform. The ratio of the speeds of the uniform motion of the trains is equal to 2. After having made a quarter of the distance between A and B, the trains

came alongside, at the moment the speed of the one being $1,5$ times higher than the speed of the other. Find the ratio of periods of time, during which the trains have passed the distance between A and B.

Answer: $\frac{2}{3}$.

14. In a 7×7 square (composed of square cells), it is necessary to mark centers of k cells so that any four marked points would not be the vertices of the rectangle with the sides, parallel to the sides of the square. For what greatest k is it possible?

Answer: 21.

15. The matrix with all entries either 0 or 1 is called a binary matrix. What is the greatest possible number of units in a invertible binary 10×10 matrix?

Answer: 91.

16. Square matrix A is called orthogonal, if $A^T A = E$, where E is the identity matrix. Find the sum of squares of all 2×2 minors of an orthogonal matrix of size 20×20.

Answer: 190.

17. Let $f(x,y,z) = 2x^2 + 2y^2 - 2z^2 + \frac{7}{xy} + \frac{1}{z}$. Calculate the real number n, if it is known that $n = f(a,b,c) = f(b,c,a) = f(c,a,b)$ for some pairwise different real numbers a, b, c.

Answer: 98.

18. Find the least area of the ellipse circumscribed about an isosceles triangle with the base $9\sqrt{3}$ and the altitude 5.

Answer: 30π.

19. Find the limit $\lim\limits_{x \to 0} \frac{1-(\cos x)^{\sin x}}{x^3}$.

Answer: $\frac{1}{2}$.

20. Find the limit $\lim\limits_{n \to \infty} \left(\sum_{k=1}^{n} \frac{1}{C_n^k} \right)^n$.

Answer: e^2.

21. Let $x_0 = 2014$, $x_1 = 2015$, a $x_n = \left(1 - \frac{1}{n}\right) x_{n-1} + \frac{1}{n} x_{n-2}$, $n = 2,3,\ldots$. Find $\lim\limits_{n \to \infty} x_n$.

Answer: $2015 - \frac{1}{e}$.

22. Evaluate the integral $\int\limits_{-1}^{1} x^{2013} \cdot \ln(1+e^x)dx$.

Answer: $\frac{1}{2015}$.

23. Let $t = f(x)$ be the solution of the equation $t^5 + t = x$, $x > 0$. Evaluate the integral $\int\limits_0^2 f(x)dx$.

Answer: $\frac{4}{3}$.

24. $y = \frac{1}{x^2 - 20x + 99}$. Find $y^{(8)}(10)$.

Answer: -40320.

25. Evaluate the integral $\int\limits_0^{+\infty} \frac{\operatorname{arctg} 3x - \operatorname{arctg} 9x}{x} dx$.

Answer: $-\frac{\pi \ln 3}{2}$.

26. Find the volume of a solid bounded by the paraboloid of revolution $z = x^2 + y^2$ and the plane $z = x + y$.

Answer: $\frac{\pi}{8}$.

27. The air in a room with the volume of 200 m^3 contains $0{,}15\%$ of the carbon dioxide CO_2 . A mechanical fan blows in 20 m^3 of air, containing $0,04\%$ CO_2 , per minute. In what time (in minutes) will the amount of carbon dioxide in the room air decrease by three times?
Answer: $10 \ln 11$.

28. Find the sum of the series $\sum\limits_{n=0}^{\infty} \frac{4^n (2n+1)}{n!}$.

Answer: $9e^4$.

29. Let x_1 , x_2 , …, x_{2014} be the roots of the equation $x^{2014} + x^{2013} + \ldots + x + 1 = 0$. Find $\sum\limits_{k=1}^{2014} \frac{1}{1 - x_k}$.

Answer: 1007 .

30. Evaluate the integral $\int\limits_0^{+\infty} \frac{x^2 - 4}{x^2 + 4} \cdot \frac{\sin 2x}{x} \cdot dx$.

Answer: $\frac{\pi}{e^4} - \frac{\pi}{2}$.