1. One morning, at 25 minutes past nine, a pedestrian set out from point A to point B. Walking at a constant speed, he arrived at point B at a quarter past one that same day. The next day, at 11 o'clock in the morning, he started back from B to A. Walking uniformly, but somewhat faster than the day before, he arrived at point A at twenty minutes to three. Knowing that the distance between the points is 12 kilometers, determine the distance (in kilometers) from A to the place that the pedestrian passed at the same time on each of these days.
Answer: $8,4$.
2. Find the sum of all real solutions (or the solution, if there is only one) of the equation

$$64\left(1+\frac{4}{x-1}\right)^3-\left(1+\frac{1}{x+2}\right)^3=63.$$

Answer: $-\frac{11}{5}$.
3. Find the smallest real solution of the equation $\sqrt{1-x^2}=4x^3-3x$.
Answer: $-\frac{\sqrt{2}}{2}$.
4. Find the sum of the real solutions of the equation $\sin\left(\pi\cos x\right)=\cos\left(\pi\sin x\right)$ on the closed interval $\left[0,\pi\right]$.
Answer: $2\arccos\frac{1}{2\sqrt{2}}$.
5. Find the sum of all integer solutions of the equation

$$2^{2x+2}-3\left(2x^2-4x+5\right)2^{x+1}+9x\left(x^3-4x^2+9x-10\right)+56=0.$$

Answer: 10.
6. Find the sum of all real solutions of the equation $\left|1-\lg x\right|+\left|1+\lg x\right|=4\left(1-\frac{\left|x-5\right|}{5}\right)$.
Answer: 10.
7. Find the arithmetic mean of all real solutions of the equation $x=\sum_{n=2}^{2026}\left[\frac{x}{n}\right]$, where $\left[x\right]$ denotes the greatest integer not exceeding x.
Answer: 3.
8. It is known that in some arithmetic progression of natural numbers, the product of the second and third terms is equal to the tenth term. Find the largest possible value of its fifth term.
Answer: 13.
9. Find a six-digit natural number (in the decimal system) such that when multiplied by 2, 3, 4, 5, and 6 it yields numbers written with the same digits as the original number, but in a different order.
Answer: 142857.
10. How many distinct ways are there to represent one million as a product of three natural numbers? Representations that differ only in the order of the factors are considered identical.
Answer: 139.
11. An 8×8 square, divided into unit squares, was partitioned into rectangles consisting of two cells. Two rectangles in the partition are called neighboring if they share a common boundary segment of length one or two. In each rectangle, we write the number of rectangles neighboring it. Find the largest possible value of the sum of all written numbers.
Answer: 158.
12. In a right-angled trapezoid $ABCD$, the legs AB and CD are 8 cm and 17 cm, respectively. A line parallel to the bases divides the trapezoid into two smaller trapezoids, each of which can be circumscribed about a circle. Find the length (in cm) of the longer base of trapezoid $ABCD$.
Answer: 16.

13. Two circles with radii 5 cm and 20 cm touch each other externally at point A. Through point B, taken on the larger circle, a line is drawn that touches the smaller circle at point C. Find the length (in cm) of the segment BC given that the length of the chord AB is 30 cm.
Answer: $15\sqrt{5}$.
14. The development of the lateral surface of a cone is a sector of angle $120°$. A triangular pyramid is inscribed in the cone such that the angles of its base are in arithmetic progression with difference $15°$. Determine the angle that the smallest lateral face makes with the plane of the base.
Answer: $\arccos\frac{\sqrt{17}}{17}$.
15. The base of the triangular pyramid $SABC$ is a right isosceles triangle ABC, with $AB = BC$, $\angle ASC = 90°$, $AS = SC$. The faces ASC and ABC are mutually perpendicular. The radius of the sphere inscribed in the pyramid is $\frac{\sqrt{2}}{4+2\sqrt{3}}$. Find the distance from the center of the sphere circumscribed around the pyramid to the plane ABS.
Answer: $\frac{1}{\sqrt{6}}$.
16. Evaluate the determinant of order 2026 in which all entries on the main diagonal are 5, all entries on the anti-diagonal are 3, and all other entries are zero.
Answer: 2^{4052}.
17. What is the largest possible dimension of a subspace of the space of 9×9 matrices over the field of real numbers, that contains only singular matrices?
Answer: 72.
18. Find the distance between the foci of the ellipse defined by the equation

$$19x^2 + 49y^2 + 30\sqrt{3}xy - 256 = 0.$$

Answer: $4\sqrt{15}$.
19. The sequence (a_n) is defined by the recurrence formula $a_{n+1} = a_n + \sqrt{1+a_n^2}$, $a_1 = 1$. Find the limit $\lim\limits_{n\to\infty}\frac{2^n}{a_n}$.
Answer: $\frac{\pi}{2}$.

20. Find the limit $\lim\limits_{x\to 0}\left(\frac{1}{5}\sum\limits_{n=1}^{5}(2n-1)^x\right)^{\frac{1}{x}}$.
Answer: $\sqrt[5]{945}$.
21. Find the limit $\lim\limits_{x\to 0}\frac{\sqrt[3]{\sin x^3}-x}{x^7}$.
Answer: $-\frac{1}{18}$.

22. Find the smallest value of the function $y = \frac{(x^2-x+1)^3}{x^6-x^3+1}$ on the interval $(0,+\infty)$.
Answer: $2\sqrt{3}-3$.

23. Evaluate the integral $\int\limits_0^{\pi/2} \ln\left(4-\sin^2 x\right)dx$.

Answer: $\pi\ln\left(1+\frac{\sqrt{3}}{2}\right)$.

24. Let $y=y(x)$ be a solution to the integral equation $y(x)=x^3+\int\limits_0^x \sin(x-\xi)y(\xi)d\xi$. Find $y(2)$.

Answer: $\frac{48}{5}$.

25. A person drank a cup of filtered coffee containing 100 mg of caffeine. Caffeine is rapidly and completely absorbed into the bloodstream. The volume of distribution of caffeine in the body averages 35 L (calculated as 0.5 L/kg for a 70 kg person). The half-life of caffeine in the body is 5 hours, and its elimination is proportional to its concentration. Calculate the concentration of caffeine in the blood plasma (in mcg/mL) 3 hours after consumption. The volume of distribution is the hypothetical volume of fluid required to dissolve the entire dose of a substance in order to obtain the same concentration as that in the blood plasma.

Answer: $\frac{20}{7\sqrt[5]{8}}$.

26. Let $y=y(x)$ be a solution to the differential equation $xy'-(2x+1)y+x^2+y^2=0$, satisfying the condition $y(1)=0$. Find $y(-1)$.

Answer: $-\frac{2}{3}$.

27. A square $D=\left\{(x,y)\in\mathbb{R}^2 : 1<x<1+h,\ 4<y<4+h\right\}$ is transformed into a domain D' by the system of functions $u=\frac{y^2}{x}$, $v=\sqrt{xy}$. Find the limit of the ratio of the area of D' to the area of D as $h\to 0$.

Answer: 12.

28. Find the largest real value of the parameter p for which all solutions of the equation

$$z^3+12\left(1+i\sqrt{3}\right)z+p=0 \ (z\in\mathbb{C})$$

lie on one line.

Answer: $32\sqrt{2}$.

29. Find the sum of the series $\sum\limits_{n=0}^{\infty} 2^{\left[\frac{n}{2}\right]}\cdot 5^{-\left[\frac{n+1}{2}\right]}$. Here $[x]$ denotes the greatest integer not exceeding x.

Answer: 2.

30. A function $f(x)$ continuous on $\mathbb{R}$ is expanded in a series: $f(x)=\frac{1}{2}-\frac{4}{\pi^2}\sum\limits_{n=1}^{\infty}\frac{\cos\left((2n-1)\pi x\right)}{(2n-1)^2}$.

Additionally, it is known that $f(0)=0$. Find $f(\pi)$.

Answer: $4-\pi$.