1. During the test, two self-driving trucks are moving towards each other on a straight highway at constant speeds. The speed of the first truck is 60 km/h, the speed of the second truck is 70 km/h. When the distance between the trucks is 80 km, a diagnostic drone takes off from the first truck and moves at a speed of 170 km/h in a straight line towards the second truck. When the drone reaches the second truck, it docks for three minutes with a docking station installed on this truck, and then returns to the first truck, again moving at a speed of 170 km/h. How many minutes was the drone absent from the first truck?

Answer: $30\frac{20}{23}$.

2. Find the product of all integer values of the parameter $a \in [-7,7]$, for which the equation

$$x(x+1)(x+a)(x+1+a) = a^2$$

has four different real solutions.
Answer: -44100.

3. Find the product of all real solutions to the equation $2(x^2+2) = 5\sqrt{x^3+1}$.

Answer: -3.

4. Find the sum of the solutions of the equation $\operatorname{ctg}^4 2x - \cos^2 4x = 1$ that belong to the segment $[0,2\pi]$.
Answer: 8π.

5. Find the smallest real solution of the equation $\sqrt{e^{2x} - 0,5 + 2\sqrt{e^{2x}-1}} = e^x$.

Answer: $\ln\frac{7}{4\sqrt{3}}$.

6. Determine the number of natural numbers from the segment $[1,1000]$ that can be represented as $[n+\sqrt{n}+1/2]$ for some $n \in \mathbb{N}$. Here $[x]$ is the largest integer not greater than x.
Answer: 969.
7. In a sequence of natural numbers, each member, starting with the third, is equal to the modulus of the difference of the two previous members. What is the greatest number of members such a sequence can have if each of its members does not exceed 2025?
Answer: 3039.
8. What is the greatest number of terms (after combining like terms) that a polynomial of the hundredth degree of three variables x, y, z can contain?
Answer: 176851.
9. Each pair (x,y) of integers was assigned a certain integer and this integer was denoted by $x \circ y$. Numbers $x \circ y$ and $y \circ x$ can be different. It turned out that for any integers a, b, c and d the equality $(a \circ b + d) \circ c = (a-b) \circ (c-d) + 1$ holds. Find the value of $2025 \circ 1991$.
Answer: 34.
10. In a chess tournament, each chess player scored half of his points in games with the participants who took the last three places. How many players took part in the tournament? (The tournament was held according to the round-robin system, i.e. each participant played with every other participant; one point was awarded for a win, half a point for a draw, and zero points for a loss).
Answer: 9.
11. There are 19 points on a plane, no three of which lie on the same line. What is the greatest number of segments with ends at these points that can be drawn so that not a single triangle with vertices at these points is obtained?
Answer: 90.
12. From each vertex of the base of an equilateral triangle with a side of 6 cm, two rays are drawn, forming angles of $15°$ and $30°$ with this base. Find the area (in cm^2) of a quadrilateral whose vertices are the intersection points of the drawn rays.
Answer: $27 - 15\sqrt{3}$.

13. A convex quadrilateral with an area of 25 cm^2 is divided by diagonals into four triangles. Find the area (in cm^2) of a quadrilateral whose vertices are the intersection points of the medians of these triangles.

Answer: $\frac{50}{9}$.

14. A regular quadrangular pyramid is inscribed in a ball of radius 2 cm. The base of the pyramid divides in half the radius perpendicular to it. Determine the largest possible value of the area (in cm^2) of the surface of the ball inscribed in the pyramid.

Answer: $2\pi\left(4-\sqrt{7}\right)$.

15. The length of the lateral edge of a regular triangular pyramid is equal to the height of the base of the pyramid and is equal to one. A sphere touches the plane of the base of the pyramid, two of its lateral edges and the extension of the third lateral edge beyond the top of the pyramid. Find the radius of this sphere.

Answer: $\frac{4\sqrt{5}}{15}$.

16. Find the determinant of a tridiagonal matrix $A=\left(a_{ij}\right)$ of size 10×10, the elements of the main diagonal of which are equal to 10, and the elements of lower and upper diagonals are equal to 4 ($a_{ij}=0$, if $\left|i-j\right|>1$).

Answer: 1431655424.

17. Find the eccentricity of the second-order curve given by the equation

$$9x^2-4xy+6y^2+16x-8y-2=0.$$

Answer: $\frac{\sqrt{2}}{2}$.

18. On the edges AA_1 and CC_1 of the parallelepiped the points M and N are located respectively so that $\left|AM\right|:\left|AA_1\right|=1:4$, $\left|CN\right|:\left|CC_1\right|=4:5$. The plane P, passing through the points M and N, and parallel to the diagonal BD of the base, intersects the edge BB_1 at the point K. Find $\left|BK\right|:\left|KB_1\right|$.

Answer: $\frac{21}{19}$.

19. Find the limit $\lim\limits_{n\to\infty} n^2\left(\sqrt[n]{2025}-\sqrt[n+1]{2025}\right)$.

Answer: $2\ln 45$.

20. A function $y=f\left(x\right)$ is defined for $x>0$ and is bounded on each interval $\left(0,a\right)$. It is known that $\lim\limits_{x\to+\infty}\frac{f\left(x+1\right)-f\left(x\right)}{x^{13}}=13$. Find $\lim\limits_{x\to+\infty}\frac{f\left(x\right)}{x^{14}}$.

Answer: $\frac{13}{14}$.

21. Find the minimum of the function $f\left(x\right)=\max\left\{2\left|x\right|,\left|1+x\right|\right\}$.

Answer: $\frac{2}{3}$.

22. The snowfall began before midday and continued until evening, neither increasing nor decreasing. Exactly at midday, a team of road workers went out onto the road and began to clear the snow. In the first two hours, the workers cleared 2 km of snow from the road, and in the following two hours they cleared only 1 km. The team cleared the same volumes of snow in equal intervals of time. How long before midday (in hours) did the snowfall begin?

Answer: $\sqrt{5}-1$.

23. Calculate the integral $\int\limits_0^1 x^{-\ln x-1}\ln x dx$.

Answer: $-\frac{1}{2}$.

24. Calculate the integral $\int\limits_0^{\pi/2} \frac{dx}{1+\operatorname{tg}^{\sqrt{5}} x}$.

Answer: $\frac{\pi}{4}$.

25. Let $y=y_1(x)$, $y=y_2(x)$, $y=y_3(x)$ be solutions of the differential equation

$$y'''+p(x)y''+q(x)y'+r(x)y=0$$

such that $y_1^2(x)+y_2^2(x)+y_3^2(x)=1$, $x\in\mathbb{R}$. Let $f(x)=\left(y_1'(x)\right)^2+\left(y_2'(x)\right)^2+\left(y_3'(x)\right)^2$ and let a and b be real constants such that $y=f(x)$ is a solution of the differential equation $y'+ap(x)y=br(x)$. Find the sum of constants a and b.

Answer: $\frac{4}{3}$.

26. Let $y=y(x)$ be the solution to the equation $\int\limits_0^x \operatorname{sh}(x-t)y(t)dt=x-y$. Find $y(1)$.

Answer: $\frac{5}{6}$.

27. Calculate the volume of the body bounded by the surface, which is given by the equation

$$\left(x^2+y^2+z^2\right)^2=4\left(x^2+y^2-z^2\right).$$

Answer: $\sqrt{2}\pi^2$.

28. Find the length of the spatial curve given by equations $x^2+y^2=z$, $\frac{y}{x}=\operatorname{tg} z$, from the origin of coordinates to the point with applicate $z=\frac{\pi}{4}$.

Answer: $\frac{\sqrt{\pi}(\pi+6)}{12}$.

29. Find the sum of the series $1+\frac{1}{3}-\frac{1}{5}-\frac{1}{7}+\frac{1}{9}+\frac{1}{11}-\frac{1}{13}-\frac{1}{15}+\dots$.

Answer: $\frac{\pi\sqrt{2}}{4}$.

30. For what largest integer value of the parameter a does the series $\sum\limits_{n=1}^{\infty}\left(n^{n^a}-1\right)$ converge?

Answer: -2.