1. What is the maximum distance (in kilometers) that three people with one two-seater electric scooter can cover in three hours? Pedestrian speed is 5 km/h, electric scooter speed (without or with load) is 30 km/h, maximum distance on a single charge is 90 km.

Answer: $\frac{810}{19}$.

2. Find the smallest real value of a, for which there exist four natural numbers (x, y, u, v) satisfying the equalities

$$\begin{cases}(x+y)(x+y+20)=(140-a)(a-80),\\ a\left(8u^2+2v^2-a\right)=\left(4u^2-v^2\right)^2.\end{cases}$$

Answer: 100.

3. Find the largest real solution to the equation

$$\sqrt[5]{(x-2)(x-32)}-\sqrt[5]{(x-1)(x-33)}=1.$$

Answer: $17+\sqrt{257}$.

4. What is the smallest real value of a, for which the inequality $\sin^6 x+\cos^6 x+a\sin x\cos x\geq 0$ holds for all real x?

Answer: $-\frac{1}{2}$.

5. Find the product of solutions to the equation $(3x)^{3\log_6 2x-4}=2024\cdot x^{\log_6 x}$.

Answer: $\sqrt{6}$.

6. The function $f(n)$ is defined for all natural n and takes non-negative integer values. It is known that $f(n)$ satisfies the conditions: a) $f(m+n)-f(m)-f(n)$ takes values 0 or 1 for any m and n; b) $f(2)=0$; c) $f(3)>0$; г) $f(9999)=3333$. Find $f(2024)$.

Answer: 674.

7. The function $f:\mathbb{R}\to\mathbb{R}$ for all real values of the variables x and y satisfies the equality $f(x-0{,}25)+f(y-0{,}25)=f(x+[y+0{,}25]-0{,}25)$, where $[x]$ is the largest integer not greater than x. It is also known that $f(3)=4$. Find $f(5)$.

Answer: $\frac{20}{3}$.

8. When a sheet of paper is rotated in its plane by 180°, the designations of numbers 0, 1, 8 do not change, the designations of 6 and 9 transform into each other, and the recording of the remaining numbers becomes meaningless. Among the seven-digit numbers, the writing of which does not change when the sheet of paper is rotated by 180°, how many are divisible by four?

Answer: 75.

9. Let $S=\{1,2,3,\ldots,280\}$. Find the smallest natural number n such that any n-element subset of the set S contains five pairwise coprime numbers.

Answer: 217.

10. Find the largest number of areas into which a disk is cut by segments connecting nine points lying on its circle.

Answer: 163.

11. Given a paper strip 1×2024. A and B play the game by taking turns. A starts. In one move, it is allowed to paint any not yet painted cell of the strip in any of two colors - red or blue. When all the cells of the strip are painted, it is cut into the minimum possible number of strips so that all the cells of each strip are the same color. What is the maximum number of such strips that B can guarantee, regardless of actions of A?

Answer: 1013.

12. A disk is inscribed in a triangle ABC with sides of 3 cm, 5 cm и 7 cm and tangents to it are constructed parallel to the sides of this triangle. These tangents cut off three new triangles from this triangle

ABC. A disk is inscribed in each of the triangles constructed in this way. Calculate the sum of the areas (in cm^2) of all four disks.

Answer: $\frac{83\pi}{75}$.

13. Chords AB and CD intersect at point E inside a given circle. Let M be the interior point of the segment BE, $AM : AB = 5 : 7$. The tangent at point E to the circle passing through points D, E and M, intersects the lines BC and AC at points F and G, respectively. Find $EG : EF$.

Answer: $\frac{5}{2}$.

14. The base of the pyramid $SABC$ is the equilateral triangle ABC. Edge SA is perpendicular to the plane of the base. Find the angle between the side face SBC and the plane of the base if the side surface of the pyramid relates to the area of the base as $11 : 4$.

Answer: $\operatorname{arctg}\frac{3}{4}$.

15. The height of the cylinder is 3. Inside the cylinder there are three spheres of unit radius so that each sphere touches the other two and the side surface of the cylinder (that is, it has one common point with this surface). Two spheres touch the lower base of the cylinder, and the third sphere touches the upper base. Find the radius of the cylinder.

Answer: $\frac{3\sqrt{2}+4}{4}$.

16. Compute the determinant of the eighth order with the elements $a_{ii} = 5$, $a_{ij} = 2$ for $i \neq j$.

Answer: 41553.

17. Vectors $\overrightarrow{OA_1}$ and $\overrightarrow{OB}$ represent the numbers 1 and i, respectively. From point O a perpendicular OA_2 is dropped to A_1B; from A_2 perpendicular A_2A_3 is dropped to OA_1; from A_3 perpendicular A_3A_4 is dropped to A_1A_2 and so on according to the next rule: from A_n perpendicular A_nA_{n+1} is dropped to $A_{n-2}A_{n-1}$. Find the length of the limit of the vector sum $\overrightarrow{OA_1} + \overrightarrow{A_1A_2} + \overrightarrow{A_2A_3} + \ldots$.

Answer: $\frac{\sqrt{10}}{5}$.

18. Find the number of diagonals of an eight-dimensional cube that are orthogonal to a given diagonal.
Answer: 35.

19. Find the limit $\lim\limits_{n\to\infty}\left(\frac{1}{\sqrt{n^4+1}} + \frac{2}{\sqrt{n^4+2}} + \ldots + \frac{n}{\sqrt{n^4+n}}\right)$.

Answer: $\frac{1}{2}$.

20. Let the sequence (a_n) be such that for any $n \in \mathbb{N}$ $a_n \neq 1$ and $\lim\limits_{n\to\infty} a_n = 1$. Find the limit

$$\lim_{n\to\infty}\frac{a_n + a_n^2 + a_n^3 + \ldots + a_n^{100} - 100}{a_n - 1}.$$

Answer: 5050.

21. Find the least possible a such that for each quadratic trinomial $f(x)$ satisfying the condition $|f(x)| \leq 1$ for $0 \leq x \leq 1$, the inequality $f'(0) \leq a$ holds.

Answer: 8.

22. Let the function $f : [1, +\infty) \to [1, +\infty)$ be such that $f(x) = y$, where $y \geq 1$ is the unique solution to the equation $y^y = x$. Calculate the definite integral $\int_0^e f(e^x)\,dx$.

Answer: $\frac{3e^2-1}{4}$.

23. Operator $A: L_1[0,1] \to C[0,1]$ is given by the formula $(Ax)(t) = \int_0^1 (e^t + e^{-s})x(s)\,ds$. The norm of the operator A is $\|A\| = \sup_{\|x\|\le 1}\|Ax\| = \sup_{x\ne 0}\frac{\|Ax\|}{\|x\|}$; here $\|x\| = \|x\|_{L_1[0,1]} = \int_0^1 |x(s)|\,ds$, $\|Ax\| = \|Ax\|_{C[0,1]} = \max_{t\in[0,1]}|(Ax)(t)|$. Find the norm of A.

Answer: $e+1$.

24. After a lecture, the air in a 10800 m^3 classroom contains $0,12\%$ of CO_2. How many cubic meters of air, containing $0,04\%$ of CO_2, must be delivered into the classroom every minute so that 10 minutes after the break the carbon dioxide content in the audience is $0,06\%$?

Answer: $2160\ln 2$.

25. Let $y = y(x)$ be the solution to the differential equation $(x^4 - y^2)dx + xydy = 0$, that satisfies the conditions $y(1/2) = \sqrt{3}/2$, $y(1) > 0$. Find $y(1)$.

Answer: $\frac{3}{2}$.

26. Function $y = f(x) \ne 0$ satisfies the integral equation $\int_0^x f^{2024}(t)\,dt = f^{2025}(x)$. Calculate $\int_0^{2025} f(x)(52 - 3f(x))\,dx$.

Answer: 50625.

27. Find the area of the part of sphere $x^2+y^2+z^2=1$, bounded by parallels $\psi_1 = 45°$, $\psi_2 = 60°$ and meridians $\varphi_1 = 34°$, $\varphi_2 = 37°$.

Answer: $\frac{(\sqrt{3}-\sqrt{2})\pi}{120}$.

28. Calculate $\int_0^{+\infty}\int_0^{+\infty} |\ln x - \ln y| e^{-(x+y)}\,dxdy$.

Answer: $2\ln 2$.

29. Let us denote by A the set of all natural numbers whose decimal notation does not contain zeros. Find the largest real value of α, for which the series $\sum_{n\in A}\frac{1}{n^\alpha}$ diverges.

Answer: $\lg 9$.

30. Find the radius of convergence of the power series $\sum_{n=1}^{\infty} e^{\int_1^{n+1}\ln[x]dx}\left(\frac{x}{n}\right)^n$, where $[x]$ is the largest integer not greater than x.

Answer: e.