1. The passenger, who missed the train, at first decided to catch up with it by taxi, but after some time he boarded the bus, paying 20 rubles for the ticket, and arrived at one of the stations at the same time as the train. Meanwhile, he discovered that if he had continued to go by taxi, he would have caught up with the train half an hour earlier, while spending 5 rubles less. What is the speed of the train (in kilometers per hour), if the taxi speed is 100 kilometers per hour, the bus speed is 80 kilometers per hour, and the fare for 1 kilometer in a taxi is 50 kopecks.
Answer: 68.
2. Find the smallest integer value of the parameter p, for which the number of integer solutions of the inequality $4x^2 - 20(x-1) + 3|4x - p| - p \leq 0$ is maximum.
Answer: 13.
3. Find the sum of the solutions to the equation $(\operatorname{tg} x + \sin x)^{1/2} + (\operatorname{tg} x - \sin x)^{1/2} = 2\operatorname{tg}^{1/2} x \cos x$, that belong to the segment $[0, 2\pi]$.
Answer: $\frac{19\pi}{6}$.
4. Find the sum of all real solutions to the equation $\frac{\lg|x^4 + 2x^3 + 2x - 1|}{\lg|x^2 + x - 1|} = 2$.
Answer: -6.
5. What is the largest n for which there are n seven-digit numbers that are consecutive members of the same geometric progression?
Answer: 11.
6. Find the largest natural number n that simultaneously satisfies two conditions: 1) the number n has at least three different natural divisors (including 1 and n itself); 2) the sum of the two largest divisors of the number n is 30 times greater than the sum of the three smallest divisors.
Answer: 775.
7. There are 12 seats in the first row of the theater. In how many ways can the holders of tickets for these seats take seats so that everyone should be either in their own seat (according to the purchased ticket) or in the neighboring one?
Answer: 233.
8. What is the smallest number of weights needed to be able to weigh any integer number of grams from 1 to 100 on balance scales if the weights can be placed in both pans?
Answer: 5.
9. The checkered strip 1×15 is numbered with the numbers 0, 1, …, 14. Two players take turns moving the chip, which is located on one of the cells, to the left by 1, 2 or 3 fields. The one who has nowhere to go loses. Under what initial positions of the chip, with the best strategy for both players, does the second player win? Write in the answer the sum of the numbers of the corresponding cells.
Answer: 24.
10. The city has the shape of a square with a side of 5 km. The streets divide it into quarters in the form of squares with a side of 200 m. What is the largest area (in square kilometers) that can be enclosed inside a closed 10 km long route along the streets of the city?
Answer: $6{,}24$.
11. A circle of radius 4 with center at point O is divided by points A, B, C, D, E, F into six equal parts. Determine the area of the figure COE, bounded by the arc OC with center at point B, arc OE with center at point F and arc CE with center at point A.
Answer: $\frac{8\pi}{3}$.
12. The equilateral triangle DEF is inscribed in the equilateral triangle ABC; point D lies on the side BC, point E lies on the side AC, and point F lies on the side AB. Ratio of the side AB to the side DF is $8:5$. Find the smallest possible value of the sine of the angle DEC.

Answer: $\frac{4\sqrt{3}-3}{10}$.

13. The length of each edge of the tetrahedron $ABCD$ is equal to 15. Points M, N and P are located on the edges DA, DC and BC respectively, so that $DM = CN = 5$, $CP = 3$. Let Q be the point of intersection of the plane MNP and line AB. Find the length of the segment BQ.

Answer: $\frac{15}{2}$.

14. The volume of the triangular prism $ABCA_1B_1C_1$ is 56. Points M and N are located on edges BB_1 and CC_1 respectively, so that $BM : BB_1 = 5 : 8$, $CN : CC_1 = 4 : 7$. Find the volume of the polyhedron $ABCA_1MN$ (truncated prism).

Answer: 41.

15. Three balls are pairwise tangent; a plane touches these balls at points A, B and C. Find the radius of the largest of the balls if the sides of the triangle ABC are $\sqrt{2}$, 2 and $1+\sqrt{3}$.

Answer: $\frac{1+\sqrt{3}}{\sqrt{2}}$.

16. Find the determinant of the ninth order with numbers 3 on its main diagonal, numbers 1 under its main diagonal, and numbers 5 over its main diagonal.

Answer: 768.

17. What is the least real value of λ for which the polynomials $P(x) = x^3 - \lambda x + 2$ and $Q(x) = x^2 + \lambda x + 2$ have a common root?

Answer: -1.

18. The base of the pyramid $SABCD$ is a parallelogram. The plane P cuts from the side edges SA, SB, SC respectively $\frac{1}{3}, \frac{1}{4}, \frac{1}{5}$ (counting from the apex S). What part does this plane cut from the edge SD?

Answer: $\frac{1}{4}$.

19. The sequence (a_n) satisfies for any natural n the relation $a_{n+2} = \frac{a_{n+1}+1}{a_n}$. Find a_{2023}, if $a_{19} = 19$, $a_{97} = 97$.

Answer: 1842.

20. The sequence (a_n) is given by the relations $a_1 = 5$, $a_{n+1} = a_n^2 - 2$, $n \geq 1$. Find the limit $\lim\limits_{n\to\infty} \frac{a_{n+1}}{a_1 a_2 \ldots a_n}$.

Answer: $\sqrt{21}$.

21. The function $f(x) \in C^2[0,1]$ has at least two zeros on the segment $[0,1]$ (counting multiplicities) and moreover $|f''(x)| \leq 1$ for all $x \in [0,1]$. What is the largest possible value of $\max\limits_{x\in[0,1]} |f(x)|$?

Answer: $\frac{1}{2}$.

22. Calculate $\int\limits_A \frac{f'(x)}{1+f^2(x)}dx$, где $f(x) = \frac{(x+1)^2(x-1)}{x^3(x-2)}$, $A = [-1,3] \setminus \{\{0\} \cup \{2\}\}$.

Answer: $\operatorname{arctg}\frac{32}{27} - 2\pi$.

23. Find the area of the figure, bounded by the curve γ, which is given by the equation $x^7 + y^7 = x^3y^3$.

Answer: $\frac{1}{14}$.

24. All living organisms are constantly involved in carbon exchange, receiving carbon from the environment. When the organism dies, it stops exchanging carbon with its environment, and the radioactive isotope ${}^{14}C$ in the remains undergoes gradual radioactive decay. The isotope ${}^{14}C$ decay rate, which describes the change in the isotope concentration per unit time, in accordance with the law of mass action, at each moment of time is directly proportional to its current concentration. The time required for a quantity of ${}^{14}C$ to reduce to half of its initial value is $5,7 \pm 0,03$ thousand years. During archaeological excavations, a tree was found, the content of ${}^{14}C$ in which was $75\ \%$ of the normal. Determine the least possible age (in years) of the found tree.
Answer: $5670\left(2 - \log_2 3\right)$.

25. Let $y = y\left(x\right)$ be the solution to the differential equation $y' + \frac{y}{x} - e^{-xy}\frac{\sin x}{x} = 0$, $\frac{\pi}{2} < x < \frac{3\pi}{2}$, that satisfies the condition $y\left(\pi\right) = 0$. Find $y\left(\frac{2\pi}{3}\right)$.

Answer: $-\frac{3\ln 2}{2\pi}$.

26. Let $y = y\left(x\right)$ be the solution to the integro-differential equation

$$\int_0^x y\left(\xi\right)d\xi - \ln\sqrt{y'} - x = 0,\ x > 0,$$

that satisfies the conditions $y\left(0\right) = 0$, $\lim\limits_{x\to+\infty} y\left(x\right) = 1$. Find $y\left(10\right)$.

Answer: $\frac{10}{11}$.

27. Let $p\left(x\right) = 2x^6 + 4x^5 + 3x^4 + 5x^3 + 3x^2 + 4x + 2$, $f\left(\alpha\right) = \int_0^{+\infty} \frac{x^\alpha dx}{p\left(x\right)}$. Find the minimum point of the function $f\left(\alpha\right)$.
Answer: 2.

28. Calculate $\int_0^{+\infty} \frac{dx}{\sqrt{x}\left(1+x\right)\left(1+x^{\sqrt{2}}\right)}$.

Answer: $\frac{\pi}{2}$.

29. Calculate the sum $\left[\frac{2024}{2}\right] + \left[\frac{2025}{4}\right] + \left[\frac{2027}{8}\right] + \ldots + \left[\frac{2023+2^n}{2^{n+1}}\right] + \ldots$, where $\left[x\right]$ is the integer part of the number x, i.e. the largest integer not greater than x.
Answer: 2023.

30. Find the sum of all real solutions to the equation $-2 - \frac{7}{x} + \frac{3}{x^2} + \frac{4}{x^3} + \frac{5}{x^4} + \ldots = 0$.

Answer: $-\frac{\sqrt{21}}{2}$.