1. A man in a boat began to row against the river current. However, 4 minutes later the boat was 80 meters downstream. Having turned the boat around, he stopped to row, and while he was resting, the boat was drifted 40 meters away. Then he began to row with the flow, and the boat was moving relative to the water at the same speed as in the first 4 minutes, and it travelled 40 meters more relative to the shore. In general, 100 seconds passed after the boat had turned around. What is the slowest possible speed of the river (in meters per minute)?
Answer: 40.
2. Find the sum of solutions to the equation

$$1-x+\frac{x(x-1)}{2}-\frac{x(x-1)(x-2)}{6}+\ldots+\frac{x(x-1)(x-2)\ldots(x-9)}{10!}=0.$$

Answer: 55.
3. Find the sum (in radians) of solutions to the equation $\left|\cos x-2\sin 2x-\cos 3x\right|=1-2\sin x-\cos 2x$ that belong to the segment $[-\pi,\pi]$.

Answer: $-\frac{\pi}{2}$.
4. Find the largest value of the parameter a, for which every solution of the inequality

$$\left(\frac{1}{2}\right)^{\frac{1}{2(x-1)^2}}\le\left(\frac{1}{4}\right)^{\frac{1}{(3-x)^2}}$$

is the solution of the inequality $16a^4x^2-9\le 0$.
Answer: $\frac{3\sqrt{5}}{10}$.
5. How many numbers are simultaneously the members of the following arithmetic progressions: 3, 7, 11, …, 407 and 2, 9, 16, …, 709?
Answer: 14.
6. Let r be the remainder from the division of a prime number p by 210. Find r, if it is known that r is a composite number, that can be written as the sum of squares of two natural numbers.
Answer: 169.
7. How many six-digit numbers are there whose sum of digits is equal to six (the first digit is different from zero)?
Answer: 252.
8. What is the largest number of colors that can be used to color a 8×8 board in such a way that each square has the common side with at least two squares of its own color? Each square is entirely painted one color.
Answer: 16.
9. The tic-tac-toe (noughts and crosses) game is played up to the moment when three marks (three O's or three X's) are lined up in a row. What is the smallest number of squares, which a grid (of an arbitrary shape) must have, so that the player, who begins, could win, no matter how his opponent plays?
Answer: 7.
10. Eleven players play the following game: there are 23 checkers, 12 of them are black, and the rest are white. The players sit around the table and each of them receives two checkers. By lot, one of the players takes the remaining checker. If it turns out that he has three checkers of the same color, then he wins the game, and the game stops; if not, he keeps the checkers of the same color for himself, and gives the third one to the neighbor on the right. If his neighbor turns out to have three checkers of the same color, then he wins, if not, he does what the first player has done, etc. Find the maximum possible number of the moves in this game.
Answer: 16.
11. The bases of the heights of an acute angled triangle ABC serve as the vertices of another triangle whose perimeter is equal to 10. Find the area of the triangle ABC if the radius of the circumscribed circle of this triangle is equal to 7.
Answer: 35.

12. When rotated with the center at the point C at an angle of $15°$, the triangle ABC becomes a triangle $A'B'C$. The point B' is the image of the point B and it lies on the side AB; the point A' is the image of the point A. Find the smaller angle (in degrees) of the triangle ABC if the straight lines AC and $A'B'$ are perpendicular.
Answer: $22,5$.

13. From the middle of the height of a regular quadrangular pyramid, a perpendicular of length $\sqrt{6}$ to the lateral edge and a perpendicular of length 2 to the lateral face are drawn. Find the volume of the pyramid.
Answer: $128\sqrt{3}$.

14. The length of edge of the cube $ABCDA_1B_1C_1D_1$ is 3. The points M and Q are taken on the edges AD and B_1C_1, respectively, and the points P and N are taken on the edge CD so that $|AM| = |C_1Q| = |CP| = |DN| = 1$. Find the area of the cross-section of the cube formed by a plane passing through the straight line MP parallel to the straight line NQ.

Answer: $\frac{13\sqrt{3}}{2}$.

15. Let the sphere $S \subset \mathbb{R}^3$ intersect the sphere B of radius 1 and pass through its center. Find the area of the part of the sphere S that is contained inside the sphere B.
Answer: π.

16. Find the determinant of the tenth order with numbers 2023 on its main diagonal and with all other elements equal to 2022.
Answer: 20221.

17. Let vectors $\vec{a}$ and $\vec{x}$ be not orthogonal and $|\vec{a}| = 1$. How many different vectors are there in the sequence $\vec{x}_0 = \vec{x}$, $\vec{x}_1 = \vec{a} \times \vec{x}_0$ (cross product of vectors $\vec{a}$ and $\vec{x}_0$), $\vec{x}_2 = \vec{a} \times \vec{x}_1, \ldots$?
Answer: 5.

18. Find the shortest distance between the surface $4z = x^2 + y^2$ and the plane

$$2x - y + 2z + 3 = 0.$$

Answer: $\frac{1}{6}$.

19. The sequence a_n is given by the equalities

$$a_1 = \sqrt{13},\ a_2 = \sqrt{13 + \sqrt{13}},\ a_3 = \sqrt{13 + \sqrt{13 + \sqrt{13}}},\ \ldots.$$

Find $\lim\limits_{n\to\infty} a_n$.

Answer: $\frac{1 + \sqrt{53}}{2}$.

20. Find the limit $\lim\limits_{n\to\infty} \frac{(n+1)\ln(n!) - 2\ln(2!3!\ldots n!)}{n^2 + n}$.

Answer: $\frac{1}{2}$.

21. A straight section of the railway runs along the edge of the forest. The lineman is in the forest, 5 km away from this road and 13 km away from his house, standing by the railway. He can walk at a speed of 3 km/h through the forest and 5 km/h along the railway lines. What is the shortest time (in minutes) for the lineman to get home?
Answer: 224.

22. Find the area of the figure which is formed when the parabolas $x^2 = \pm 22y$, $y^2 = \pm 22(x \mp a)$ touch.

Answer: $\frac{242}{3}$.

23. Find the mass of a homogeneous plate bounded by the part of the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 4$ and by the segments $0 \le x \le 8$, $0 \le y \le 8$. The density of the plate is equal to 1.
Answer: 6π.
24. A cylindrical tank is put vertically and has a hole in the bottom. Water flows out of the tank at a speed equal to $0,6\sqrt{2gh}$, where $g = 10\,\text{m/sec}^2$ is the acceleration of gravity, h is the height of water level above the hole. Half of the water from a full tank flows out in 5 minutes. How long (in minutes) will it take for all the water to flow out?
Answer: $5\left(2 + \sqrt{2}\right)$.
25. Let $y = y\left(x\right)$ be the solution to the differential equation $\left(x^2 + 3\ln y\right)ydx = xdy$, that satisfies the condition $y\left(1\right) = 1$. Find $y\left(2\right)$.
Answer: e^4.
26. For which largest value of λ the equation $y' = ax^3 + by^\lambda$ is reduced to a homogeneous equation by using the substitution $y = z^m$?
Answer: $\dfrac{3}{4}$.
27. Let $F\left(x\right)$ be the Weierstrass transform of the function $f\left(t\right) = \cos t$:

$$F\left(x\right) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{+\infty} e^{-\left(x-t\right)^2} \cos t\,dt.$$

Find $F\left(\pi\right)$.
Answer: $-e^{-\frac{1}{4}}$.
28. Find the integral $\displaystyle\int_{-\infty}^{+\infty} \frac{e^{x/6}}{1+e^x}\,dx$.
Answer: 2π.
29. Find the sum of the series $\displaystyle\sum_{n=0}^{\infty} \frac{n^2 - 1}{3^n n!}$.
Answer: $-\dfrac{5}{9}e^{\frac{1}{3}}$.
30. Let the series that is infinite in both directions

$$\ldots + f''\left(x\right) + f'\left(x\right) + f\left(x\right) + \int_0^x f\left(t\right)dt + \int_0^x dt_1 \int_0^{t_1} f\left(t\right)dt + \ldots$$

converge uniformly to the function $g\left(x\right)$ and $g\left(0\right) = 2$. Find $g\left(2\right)$.
Answer: $2e^2$.