1. A vessel containing a 70% acid solution was filled to the brim with a 9% acid solution and after mixing, the same amount was poured out. After this operation had been repeated 5 times, a 15% solution was obtained. What part of the vessel volume did the initial solution occupy?

Answer: $\sqrt[5]{\frac{6}{61}}$.

2. Find the least integer value of k, for which the equation $\left|x^2-1\right|+kx=\left|x^2-8x+15\right|+b$ has more than 5 different real solutions for at least one real number b.
Answer: -8.

3. Determine, how many solutions does the equation $x^2-\left[x^2\right]=\left\{x\right\}^2$ have on the segment $\left[1,100\right]$. Here $\left[x\right]$ is the largest integer not greater than x, $\left\{x\right\}=x-\left[x\right]$.
Answer: 9901.

4. For which largest natural n does the quantity $\frac{\lg 2\cdot\lg 3\cdot\ldots\cdot\lg n}{10^n}$ take the least value?

Answer: 10^{10}.

5. Find the largest real value of a, for which the roots x_1, x_2, x_3 of the polynomial x^3-6x^2+ax+a satisfy the equality $\left(x_1-3\right)^3+\left(x_2-3\right)^3+\left(x_3-3\right)^3=0$.
Answer: -9.

6. Find the largest solution of the equation $\operatorname{ctg}^4 2z=\cos^2 4z+1$, which belongs to the segment $\left[0,\pi\right]$.

Answer: $\frac{7\pi}{8}$.

7. A natural number n is a square and does not end with zero. After crossing out the last two digits of this number, we again obtain the perfect square. Find the largest number n having this property.
Answer: 1681.

8. Find the largest integer A, such that for any permutation of the natural numbers from 1 to 100, the sum of some 10 consecutive numbers is greater than or equal to A.
Answer: 505.

9. What is the largest number of strips 1×6 that can be cut along the lines of the grid from the sheet of the squared paper 27×34?
Answer: 152.

10. In some country there are 30 cities and each city is connected to every other city by the road. If a road is closed for repairs, then it is impossible to take this road to go either there or back. What is the largest number of roads that can be closed for repairs so that it would be possible to drive from each city to every other city?
Answer: 406.

11. Find the number of different paths of length 10 going from the origin of coordinates to the point $\left(6,4\right)$ and consisting of segments parallel to the coordinate axes, provided that the ends of the segments are the points with integer coordinates.
Answer: 210.

12. The lengths of the four arcs, into which the entire circle with the radius 3 is divided, form a geometric progression with a common ratio equal to 3. The points of division serve as the vertices of a quadrilateral inscribed in this circle. Find the area of this quadrilateral.

Answer: $\frac{9\sqrt{2}}{4}$.

13. In the triangle ABC of the unit area, a segment AD is drawn that intersects the median CF at the point M, with $FM = \frac{1}{4}CF$. Find the area of the triangle ABD.

Answer: $\frac{2}{5}$.

14. The base edge of the regular pyramid $SABCD$ has a length of 4, and its lateral edge has a length of $2\sqrt{6}$. Find the area of the cross section of the pyramid which is perpendicular to the lateral edge SC and passes through the middle point of this edge.

Answer: $4\sqrt{6}$.

15. The edge of a regular tetrahedron $ABCD$ is equal to 8. A sphere is constructed on the edge AB as on a diameter. Find the radius of a sphere inscribed in the trihedral angle of the tetrahedron with a vertex at the point A and tangent to the constructed sphere.

Answer: $\sqrt{6} - 1$.

16. How many nonzero members does a fifth-order determinant have, whose all diagonal elements a_{11}, a_{22}, ..., a_{55} are equal to zero and all other elements are nonzero?

Answer: 44.

17. A complex number z satisfies the condition $\left|z + \frac{1}{z}\right| = 1$. What is the largest value that $\left|z\right|$ can take?

Answer: $\frac{\sqrt{5}+1}{2}$.

18. Find the ratio of the sum of the squares of the lengths of the medians of a triangle to the sum of the squares of the lengths of its sides.

Answer: $\frac{3}{4}$.

19. The angle between the axis of the right circular cone and its generatrix is equal to 30°. A plane is drawn through a certain point of the generatrix perpendicular to it. Find the eccentricity of the ellipse obtained in the cross section.

Answer: $\frac{1}{2}$.

20. Find the limit $\lim\limits_{x\to 0} \frac{\left(\sqrt{1+x^2}+x\right)^{2020} - \left(\sqrt{1+x^2}-x\right)^{2020}}{x}$.

Answer: 4040.

21. The sequences $\left(a_n\right)$ and $\left(b_n\right)$ are given as follows: $a_{-1} = 0$, $b_{-1} = 1$, $a_0 = b_0 = 1$, $a_n = 2a_{n-1} + \left(2n-1\right)^2 a_{n-2}$, $b_n = 2b_{n-1} + \left(2n-1\right)^2 b_{n-2}$, when $n \geq 1$. Find $\lim\limits_{n\to\infty} \frac{a_n}{b_n}$.

Answer: $\frac{\pi}{4}$.

22. The derivatives of the functions $f\left(x\right)$, $g\left(x\right)$, $f\left(x\right)/g\left(x\right)$ at the point $x = a$ are equal to each other and are nonzero. Find the upper bound for $f\left(a\right)$.

Answer: $\frac{1}{4}$.

23. Find the integral $\int\limits_0^{+\infty} \frac{\sin^4 7x - \sin^4 5x}{x} dx$.

Answer: $\frac{3}{8} \ln \frac{7}{5}$.

24. The amount of light absorbed when passing through a thin layer of water is proportional to the amount of incident light and the layer thickness. Knowing that when passing through a 2-meter thick layer of water, $\frac{1}{3}$ of the initial light flux is absorbed, find how much of it will reach a depth of 12 meters.

Answer: $\frac{64}{729}$.

25. Let $y(x)$ be the solution to the differential equation $xy^2(xy' + y) = 1$, that satisfies the condition $y(1) = 1$. Find $y(0{,}5)$.

Answer: -1.

26. Let $y(x)$ be the solution to the boundary value problem $x^2y'' - 2xy' + 2y = 0$, $y(x) = o(x)$ when $x \to 0$, $y(1) = 3$. Find $y(0{,}25)$.

Answer: $\frac{3}{16}$.

27. Find the area of the figure, bounded by the curve $\left(\frac{x}{2} + \frac{y}{3}\right)^4 = 4xy$.

Answer: 24.

28. Let a smooth plane curve Γ bound a convex domain Ω whose area is equal to 10π. A segment, having a length of 1 with the ends lying on the curve Γ, moves along the curve Γ so that its ends pass through all the points of the curve, and the middle point describes a smooth curve γ bounding the domain $G \subset \Omega$. Find the area of the domain G.

Answer: $\frac{39\pi}{4}$.

29. Find the sum of the series $\frac{1!}{2021} + \frac{2!}{2021 \cdot 2022} + \frac{3!}{2021 \cdot 2022 \cdot 2023} + \ldots$.

Answer: $\frac{1}{2019}$.

30. Find the sum of the series $\sum\limits_{n=-\infty}^{+\infty} \frac{1}{(1 + 4n)^2}$.

Answer: $\frac{\pi^2}{8}$.