1. In the pool, three swimmers should swim in a 50-meter long lane, immediately turn around at its end and swim back to the starting point. First, the first swimmer started; then, in ten seconds, the second swimmer took the start; and at last, the third one started after another ten seconds. At some point in time, before reaching the end of the lane, the swimmers were at the same distance from the starting point. The third swimmer, after reaching the end of the lane and turning around, met the second swimmer, who was still five meters away from the end of the lane, and the first swimmer, who was eight meters away from the end of the lane. Find the speed (in m/s) of the first swimmer.

Answer: $\frac{6}{7}$.

2. Solve the equation $\left(1+3+5+...+\left(2n+1\right)\right):\left(\frac{1}{2}+\frac{1}{6}+\frac{1}{12}+\frac{1}{20}+...+\frac{1}{342}\right)=342$.

Answer: 17.

3. Find the least value of x, that satisfies the equation $x-\left[\sqrt{x}\right]^2=2019$. Here $\left[x\right]$ is the largest integer not greater than x.
Answer: 1022119.

4. Solve the equation $\int_0^x\left(t^2-8t+13\right)dt=x\sin\frac{a}{x}$ for those values of $a\neq 0$, for which the equation has a solution.
Answer: 6.

5. Find the sum of the largest and smallest values of the expression $f\left(x,y\right)=y-x^2$, if $\left|x\right|+\left|y\right|\leq 13$.
Answer: -156.

6. Find the number of all natural divisors of the number 10^{999} that are not divisors of the number 10^{998}.
Answer: 1999.
7. Find the number of all convex pentagons whose vertices are 5 out of 25 vertices of a convex 25-gon, and the two adjacent vertices must be separated by at least three vertices of the 25-gon.
Answer: 630.
8. There are two piles of matches. Initially, one pile contains m matches, and there are n matches in the other, with $m>n$. The two players alternate turns taking matches from the pile. Making a move, the player takes any (non-zero) number of matches from one pile, which is a multiple of the number of matches in the other pile. The winner is the player who takes the last match from one of the piles. What is the smallest real positive value of α, for which the following statement is true: if $m>\alpha n$, then the player making the first move can secure a victory for himself?
Answer: $\frac{1+\sqrt{5}}{2}$.
9. A reconnaissance aircraft is flying in a circle with the center at the point A. The radius of the circle is 10 km, while the speed of the aircraft is 1000 km/h. At some point in time, a missile which has the same speed as the aircraft, starts from the point A. The missile is controlled so that it is always on the straight line connecting the aircraft to the point A. How many seconds will it take the missile to reach the aircraft?
Answer: 18π.
10. An isosceles trapezium, whose area is 5 sq cm, is circumscribed around a circle having the radius of 1 cm. Find the area of the quadrilateral whose vertices are the points of tangency of the circle and the trapezium.
Answer: $\frac{8}{5}$.
11. An equilateral triangle and a square with a common vertex are inscribed in a circle of radius 2. Evaluate the area of the common part of the triangle and the square.
Answer: $8\sqrt{3}-9$.

12. An edge of a cube has the length of 9. Find the distance between the straight lines on which the skew diagonals of the two adjacent faces of the cube lie.
Answer: $3\sqrt{3}$.
13. In a rectangular parallelepiped $ABCDA_1B_1C_1D_1$ the perpendiculars A_1P and BQ are dropped from the vertices A_1 and B on the diagonal AC_1. Find the length of the line segment PQ, if $AB = 4$, $AD = 3$, $AA_1 = 5$.

Answer: $\dfrac{9}{5\sqrt{2}}$.
14. A seventh-order square matrix $A = \left(a_{ij}\right)$ is defined as follows: $a_{ij} = ij$, if $i \neq j$ and $a_{ii} = i^2 + 2$. Evaluate $\det A$.
Answer: 9088.

15. Let the norm $\left\| P \right\|$ of a polynomial P be the sum of the modules of its coefficients. Find $\max\limits_{E} \dfrac{\left\| P^{(5)} \right\|}{\left\| P \right\|}$, where $E = \left\{ P \middle| \deg P = 5, P(1) = 0 \right\}$, $\deg P$ is the degree of the polynomial P, and $P^{(5)}$ is the fifth order derivative of the polynomial P.
Answer: 60.
16. Let A_1, A_2, ..., A_n be the vertices of a regular 12-gon inscribed in a circle with a radius of one, B is some point of this circle. Find $\sum\limits_{n=1}^{12} \left| BA_n \right|^2$.
Answer: 24.
17. Find the distance between the nearest points of the curve $y = x^4 + 3x^2 + 2x$ and the line $y = 2x - 1$.

Answer: $\dfrac{1}{\sqrt{5}}$.

18. Find $\lim\limits_{n\to\infty} \left(\sin\dfrac{\pi}{n} \right) \cdot \sum\limits_{k=1}^{n} \dfrac{1}{2 + \cos\dfrac{k\pi}{n}}$.

Answer: $\dfrac{\pi}{\sqrt{3}}$.

19. Let $a_{i,0} = \dfrac{1}{2^{i-1}}$, $a_{i,j+1} = a_{i,j}^2 + 2a_{i,j}$. Find $\lim\limits_{n\to\infty} a_{n,n}$.
Answer: $e^2 - 1$.
20. Let the function $y = f(x)$ be infinitely differentiable on the interval $(-a, a)$, $a > 1$, and let the sequence $\left(f^{(n)}(x) \right)$ converge uniformly on this interval. Let $\lim\limits_{n\to\infty} f^{(n)}(0) = 1$. Find $\lim\limits_{n\to\infty} f^{(n)}(1)$.
Answer: e.

21. Let $f(x+h) = \sum\limits_{k=0}^{2018} \dfrac{h^k}{k!} f^{(k)}(x) + \dfrac{h^{2019}}{2019!} f^{(2019)}(x + \theta h)$, where $0 < \theta < 1$, $f^{(2020)}(x) \neq 0$. Find $\lim\limits_{h\to 0} \theta$.

Answer: $\dfrac{1}{2020}$.

22. Find the integral $\int\limits_0^1 \frac{\ln\left(1-0,01x^2\right)}{x^2\sqrt{1-x^2}}dx$.

Answer: $\pi\left(\sqrt{0,99}-1\right)$.

23. Find $\int\limits_0^7 z^2\left(x\right)dx$, where $z\left(x\right)$ is a real-valued function, defined by the equation $z^3+xz=8$.

Answer: $\frac{31}{2}$.

24. Find the integral $\int\limits_0^1 \ln\Gamma\left(x\right)\sin\pi x dx$, where $\Gamma\left(x\right)=\int\limits_0^{+\infty} t^{x-1}e^{-t}dt$ is the gamma function.

Answer: $\frac{1}{\pi}\left(1+\ln\frac{\pi}{2}\right)$.

25. Find the least positive number α, for which $\lim\limits_{n\to\infty} n^\alpha \int\limits_0^{1/n} x^{x+1}dx=\frac{1}{2}$.

Answer: 2.

26. An opening got formed in the bottom of a cylindrical tank filled with liquid. Taking the flow rate of the liquid to be proportional to the height of its level in the tank and knowing that during the first 24 hours 10% of the contents flowed out, determine the amount of time (in days) during which one-half of the liquid flows out.

Answer: $\frac{\ln 0,5}{\ln 0,9}$.

27. Let $x^*\left(t\right)$ be the solution to the differential equation $\dot{x}\sin 2t=2\left(x+\cos t\right)$, which remains bounded, when $t\to\frac{\pi}{2}$. Find $x^*\left(\frac{\pi}{4}\right)$.

Answer: $1-\sqrt{2}$.

28. Find the surface area of a torus, given by the equations $x=\left(1+\cos\psi\right)\cos\varphi$, $y=\left(1+\cos\psi\right)\sin\varphi$, $z=\sin\psi$, $0\le\varphi\le 2\pi$, $0\le\psi\le 2\pi$.

Answer: $4\pi^2$.

29. Find the sum of the series $\sum\limits_{n=1}^{\infty}\frac{10^n}{\left(10^n-1\right)\left(10^{n+1}-1\right)}$.

Answer: $\frac{1}{81}$.

30. Evaluate the product $\prod\limits_{k=1}^{199}\left(e^{\frac{\pi k i}{100}}-1\right)$.

Answer: -200.