

1. It takes 5 seconds to paint one face of the cube. How much minimum time (in seconds) will three people paint 188 cubes? (It is assumed that two people cannot paint the same cube simultaneously.)
Answer: 1880.
2. There are 100 cubes. It is known that 75 of them have the yellow face, 85 cubes have the violet face and 80 cubes have the white face. Find the least number of the cubes that have white, and violet, and yellow faces.
Answer: 40.
3. A number has been written in each square of the 8×8 chessboard, and it indicates the number of rectangles in which this square enters. What is the sum of all the numbers that have been written?
Answer: 14400.
4. The 27 teams participated in the hockey championship held in one round. When the championship ended, it turned out that any three teams had a different number of points achieved in the matches between them (2 points were given for the victory, 1 point — for the draw, 0 — for the defeat). Find the largest possible number of draw games in this championship.
Answer: 182.
5. There are 10 boxes. Some of them had 10 boxes of smaller size placed into them and then more sets of 10 boxes were put in some of them, etc. In the end, it turned out that the number of the boxes, which contained at least one box, was 2017. How many boxes were used?
Answer: 20180.
6. How many natural numbers are there between 1 and 1000000, that are neither a perfect square nor a perfect cube nor the fourth degree?
Answer: 998910.
7. Find the number of digits in the number 125^{100} .
Answer: 210.
8. Find the smallest positive even number a , such that $a + 1$ is divided by 3, $a + 2$ is divided by 5, $a + 3$ is divided by 7, $a + 4$ is divided by 11, $a + 5$ is divided by 13.
Answer: 788.
9. The sum of two irreducible fractions with denominators 600 and 700 is considered. If this sum is represented as an irreducible fraction, which minimum value can its denominator take?
Answer: 168.
10. Find the sum of the roots of the equation $2x^2 + [x] = x^4$. Here $[x]$ is the largest integer not greater than x .
Answer: $\sqrt{1 + \sqrt{2}} - 1$.
11. Find the largest root of the equation $\{x^2\} = \{x\}^2$, which is smaller than 2017. Here $\{x\}$ is the fractional part of the number x .
Answer: $2016 \frac{4031}{4032}$.
12. Find the number of different pairs of integers (x, y) , satisfying the inequality $|x| + |y| < 100$.
Answer: 19801
13. Let $f(x) = \max_{y \in \mathbb{R}}(xy - f(y))$. Find the largest possible value of $f(11)$
Answer: $60 \frac{1}{2}$.
14. There are twenty numbers written on the blackboard: 1, 2, ..., 19, 20. Max and Min, in turn, put either + or - before any of these numbers. Min seeks to minimize the absolute value of the resulting sum. What is the sum with the largest absolute value which Max can have, regardless of Min's game?
Answer: 10.

15. How many points can be placed inside a disk with radius 2, so that one of the points would coincide with the center of the disk and the distances between every two points would be not less than 1.

Answer: 19.

16. In the right triangular prism $ABCA_1B_1C_1$ a cross section is drawn through the vertex A and the midpoints of the edges BB_1 and B_1C_1 . Find the ratio of volumes of parts, into which the cross section has split the prism (the volume of the part having the edge AA_1 to the volume of the part having the edge CC_1).

Answer: $\frac{13}{23}$.

17. Two circles with the radii of $\sqrt{5}$ cm and $\sqrt{2}$ cm intersect at the point A . The distance between the centers of the circles is 3 cm. A line is drawn through the point A and it intersects the circles at the points B and C , so that $AB = AC$. Find the length of the segment AB .

Answer: $\frac{6}{\sqrt{5}}$.

18. Find the largest value of the third order determinant in which two elements are equal to 4, and the rest are either 1 or -1 .

Answer: 25.

19. What is the largest value of the parameter a , for which the system of equations

$$\begin{cases} x^2 + y^2 + z^2 + 4y = 0, \\ x + a(y + z) = a, \end{cases}$$

has the only one solution?

Answer: 2.

20. Find the least volume of the body bounded by the coordinate planes and the tangent plane to the ellipsoid $\frac{x^2}{4} + \frac{y^2}{3} + \frac{z^2}{2} = 1$.

Answer: $3\sqrt{2}$.

21. Find the limit of the sequence (a_n) that is given as follows: $a_1 = 0$, $a_{n+1} = \frac{4a_n^2 + 1}{8}$.

Answer: $1 - \frac{\sqrt{3}}{2}$.

22. Find $\lim_{n \rightarrow \infty} \left(\prod_{m=1}^n \sum_{k=1}^m k^{10} \right)^{\frac{1}{n}} \cdot (n!)^{-\frac{11}{n}}$.

Answer: $\frac{1}{11}$.

23. Evaluate $\pi \int_{-\pi}^{\pi} \frac{dx}{1 + x^5 + \sqrt{1 + x^{10}}}$.

Answer: π^2 .

24. A 0,5 g pill has been thrown into a glass of water. The rate of dissolution of the pill is proportional to the mass of the pill. After what time will 99% of the pill get dissolved if it is known that 80% of it has dissolved in 10 minutes?

Answer: $\frac{10 \ln 0,01}{\ln 0,2}$.

25. Let $y(x)$ be the solution of the differential equation $\left(y - \frac{1}{x}\right)dx + \frac{dy}{y} = 0$ that satisfies the condition $y(1) = 1$. Find $y(5)$.

Answer: $\frac{5}{13}$.

26. What is the least positive value of the parameter p for which the boundary value problem $y'' + py = 1$, $y(0) = y(1) = 0$ has no solutions?

Answer: π^2 .

27. Find the area of a plane figure, the border of which is given by the equation $(x + y)^4 = x^2 - y^2$, $x \geq 0$, $y \geq 0$.

Answer: $\frac{1}{8}$.

28. Let $F(t) = \iiint_{x^2+y^2+z^2 \leq t^2} f(x^2 + y^2 + z^2) dx dy dz$, where f is a continuous function satisfying the condition $f(1) = e$. Find $F'(1)$.

Answer: $4\pi e$.

29. Find the sum of the series $\sum_{n=0}^{\infty} \frac{\cos\left(\frac{\pi n}{6}\right)}{n!}$.

Answer: $e^{\frac{\sqrt{3}}{2}} \cos \frac{1}{2}$.

30. Evaluate the integral $\int_0^{+\infty} \frac{x - \sin x}{x^3(x^2 + 4)} dx$.

Answer: $\frac{\pi(e^2 - 1)}{32e^2}$.