1. Let $Z(x)$ be the number of digits in a decimal representation of the positive number x. It is known that $Z([x^3]) \geq 9$ and $Z([x^4]) \leq 11$; here $[x]$ is the largest integer not greater than x. What is $Z([x^{12}])$ equal to?
Answer: 33.
2. Let $S(n)$ be the sum of even digits of the natural number n. For example, $S(2016) = 2 + 0 + 6 = 8$ Find $S(1) + S(2) + S(3) + ... + S(100)$.
Answer: 400.
3. How many different numbers are there in the sequence $\sin 4°$, $\sin 44°$, $\sin 444°$, …?
Answer: 9.
4. A homogeneous round disk is hung up in a horizontal position on the lace fastened to the disk centre O The loads of masses 4 g, 5 g и 7 g were placed at three different points respectively A, B, C on the disk edge without upsetting the disk balance. What is the angle AOB equal to?
Answer: $\arccos 0,2$.
5. In a quadrilateral $ABCD$ $\angle ABD = \angle CBD = 33°$, $\angle CAB = 64°$, $\angle CAD = 58°$. What is $\angle ACD$ equal to?
Answer: $65°$.
6. The length of the edge of a cube is equal to 1. Find the area of a cross section passing through the diagonal AD_1 of the face AA_1D_1D and midpoint M of the edge BB_1.
Answer: $9/8$.
7. A polygon is drawn on the coordinate plane. It turned out that its vertices have integer coordinates. Besides that there are 2016 points with integer coordinates inside the polygon, and 2015 such points are on its boundary (including the vertices). What is the area of the drawn polygon equal to?
Answer: $3022,5$.
8. Find the maximum number of diagonals of a regular 108-gon, intersecting at a single point, different from the center and vertices.
Answer: 5.
9. Five pairwise different natural numbers are given. It is known that some four of ten sums of these numbers by three are equal to 15, 20, 25 и 30. Find the greatest possible value of the sum of all five numbers.
Answer: 54.
10. Let x_1, x_2 be the roots of the square trinomial $p(x) = x^2 - 2x - 1$, and x_3, x_4 be the roots of the square trinomial $q(x) = x^2 - 3x - 1$. Find the least value of the expression
$q^3(x_1) \cdot p(x_3) + q^3(x_2) \cdot p(x_4)$.
Answer: $-21 - 5\sqrt{26}$.
11. An equilateral triangle that has sides equal to 25 is partitioned into 625 identical triangles by the straight lines that are parallel to its sides. One of the numbers 1 or -1 can be written in every small triangle, product of the number inside of every triangle and of all numbers in the neighbour (by the side) triangles being equal to 1. How many different ways can be used to fill out such a triangle table?
Answer: 4096.
12. What is the greatest number of balls of diameter 1 that can be placed in a $10 \times 10 \times 1$ box?
Answer: 106.
13. Several points were marked on a straight line. Let's consider all possible line segments with their ends at the marked points. One of the marked points is inside of 80 of these line segments, and another is inside of 90 line segments. How many points were marked?
Answer: 22.

14. Farmer Niels owns a goose farm. Once he figured out that if he sold 75 geese, then he would run out of goose feed 20 days later than if he did not sell them. But if he bought 100 additional geese, then he would use up the whole store of feed 15 days earlier than if he did not make this purchase. How many geese live on Niels' farm?
Answer: 300.
15. Twelve mathematicians came to the Mathematics Symposium, who were reflecting on k scientific problems. It turned out that there would be a problem to be solved for any group of five mathematicians on which none of them was reflecting, and, there would be no such problem for any group of six mathematicians. Find the minimum possible value of k.
Answer: 792.
16. Let's consider all real matrices $A=\left(a_{ij}\right)$ of order 2, squares of which are equal to a zero matrix. If $a_{11}=2016$, then what least value can the sum of squares of the antidiagonal entries of the matrix A take on?
Answer: 8128512.
17. Find the eccentricity of an ellipse obtained as the intersection of a right circular cylinder with a plane passing at the angle $36°$ to its axis.
Answer: $\left(\sqrt{5}+1\right)/4$.
18. Find $\lim\limits_{n\to\infty} \cos^n\left(4/\sqrt{n}\right)$.
Answer: e^{-8}.
19. The sequence of positive real numbers is given as follows: $a_0=1$, $a_n=2+\sqrt{a_{n-1}}-2\sqrt{1+\sqrt{a_{n-1}}}$. Compute $\sum\limits_{n=1}^{\infty} a_n 2^n$.
Answer: $2-2\ln 2$.
20. Find the greatest integer solution of the equation $\lim\limits_{n\to\infty}\dfrac{n^x-(n-1)^x}{(n+1)^{x-1}+(n+2)^{x-1}}=2016$.
Answer: 4032.
21. Find the sum $\sum\limits_{x=0}^{2016}\left\{\dfrac{2x+7}{2017}\right\}$, where $\{x\}$ is the fractional part of the number x.
Answer: 1008.
22. Let $y=kx+b$ be the equation of the tangent line at the point with abscissa $x_0=1$ to the graph of the even function $y=f(x)$, satisfying the identity
$f(2x^3-x)-4x^2\cdot f(x^2-x-1)=8x^5-8x^3-11x^2+2$. Find k^2+2b.
Answer: 6.
23. A twice differentiable function $f:(0;+\infty)\to\mathbb{R}$ is such that $\sup\limits_{x\in(0;+\infty)}\left|f(x)\right|\le 1$, $\sup\limits_{x\in(0;+\infty)}\left|f''(x)\right|\le 1$. What greatest value can $\sup\limits_{x\in(0;+\infty)}\left|f'(x)\right|$ take on?
Answer: 2.
24. It is known that $\varphi(2016)=3$, $\varphi'(2016)=7$. Find $\lim\limits_{n\to\infty}\left(\varphi\left(2016+1/n\right)/3\right)^n$.
Answer: $e^{7/3}$.

25. Let $y(x)$ be the function that satisfies the differential equation $yy''-2(y')^2=0$ and the conditions $y(1)=1$, $y(-1)=0{,}5$. Find $y(2016)$.
Answer: $-2/2013$.

26. The vessel of 1 liter capacity is equipped with two pipes and is filled with air containing 21% of oxygen by volume. Pure oxygen is slowly supplied through one tube into the vessel, and air and oxygen mixture flows out through the other. What percentage of oxygen will the vessel contain after 1 liter of gas passes through it?
Answer: $100-79/e$.

27. Evaluate the integral $\int\limits_0^1 \frac{\ln(x+1)}{x^2+1}dx$.

Answer: $\frac{\pi \ln 2}{8}$.

28. Evaluate the integral $\int\limits_{-\pi}^{\pi} \frac{4\sin 7x}{(1+2^x)\sin x}dx$.

Answer: 4π.

29. Find the area of a plane figure, the border of which is given by the equation $(x-y)^2+x^2=1$.
Answer: π.

30. Find the value of the integral $\oint\limits_{\gamma}\left(x\cos\left(\widehat{\vec{n},\vec{i}}\right)+y\cos\left(\widehat{\vec{n},\vec{j}}\right)\right)dl$, where γ is a simple closed curve, bounding a finite region of unit area and $\vec{n}$ is the outer normal to γ ($\vec{i}$, $\vec{j}$ are unit vectors of the coordinate axis).
Answer: 2.